Determination of optical model parameters for the elastic scattering of ²⁶Mg on ¹³C

M. Dag, M. McCleskey, R. Chyzh, V. Z. Goldberg, B. Roeder, A. Saastamoinen, E. Simmons, A. Spiridon, and R. E. Tribble

In an experiment carried out at the Cyclotron Institute, a ²⁶Mg beam at 12 MeV/u impinged on a ¹³C target and the elastic scattering and transfer reaction angular distributions were measured at several spectrometer angles as shown below in Fig 1. The elastic scattering has been used to extract the parameters of the optical model potentials that were needed for DWBA calculations to determine the ANC for the ²⁶Mg + n \rightarrow ²⁷Mg systems from the ¹³C(²⁶Mg,²⁷Mg)¹²C reaction. See Ref. [1] for more details.



FIG. 1. The elastic scattering angular distribution measurements of ${}^{26}Mg+{}^{13}C$ at several spectrometer angles. Due to consistency checks of the data at all angles, overlapping measurements were performed.

Due to the difficulties in particle identification and the uncertainty of the separation of ${}^{26}Mg_{(g,s)}$ from impurities while analyzing the elastic scattering measurements at 9.1° and 12°, those angles were excluded from the calculation of optical model parameters to eliminate an inconsistency in angular distribution measurement of ${}^{26}Mg$ on ${}^{13}C$ and to minimize the complexity of fitting procedure. Upgrading the existing setup of the Oxford Detector to have better resolution in particle identification at higher masses A≥26 is now in progress. A detailed description can be found in Ref. [2]. New measurements will be carried out after these upgrades.

In order to determine the ANC from the ${}^{13}C({}^{26}Mg,{}^{27}Mg){}^{12}C$ transfer reaction, a good and reliable optical potential is needed. The Woods-Saxon optical-model parameterization of the elastic scattering data therefore was performed with the computer code PTOLEMY ^[4]. A Woods-Saxon optical potential of form ^[3]

$$U(r) = -V_0 \left\{ \frac{1}{\left(1 + e^{r - \frac{\delta_W}{\delta_v}}\right)} \right\} - tW_0 \left\{ \frac{1}{\left(1 + e^{r - \frac{\delta_W}{\delta_w}}\right)} \right\} + Vc(r) \qquad r = r \left(A_1^{\frac{1}{2}} + A_p^{\frac{1}{2}}\right)$$

where a determines the surface diffuseness, R is the nuclear radius, V and W are related to the real and the imaginary well-depth of the potential and an additional Coulomb potential (w_0) for charged particles.

$$V_{e}(r) = \begin{cases} \frac{Z_{1}Z_{2}e^{2}}{2R_{e}} \left(3 - \frac{r^{2}}{R_{e}^{2}}\right), & r \leq R_{e} \\ \frac{Z_{1}Z_{2}e^{2}}{r}, & r > R_{e} \end{cases}$$

$$I = R_{e} = r_{ee} \left(A_{t}^{\frac{1}{2}} + A_{p}^{\frac{1}{2}}\right)$$

was used. The imaginary surface and spin-orbit interactions were neglected to decrease the number of parameters and complication of the fitting procedure.

The Woods-Saxon optical-model parameter set which best described each set of elastic scattering data was determined according to the following procedure. A grid search was performed, stepped in 1 MeV increments, over a range from 1 to 300 MeV in V and W, the depth of real and imaginary potential respectively, and then a complete search on all six parameters was run to determine the best fit. All the sets of parameters are shown in Table I.

Pot.	V [MeV]	r _v [fm]	r _w [fm]	W [MeV]	a _v [fm]	a _w [fm]	χ2	Jv [MeV fm3]	Jw [MeV fm3]	Rv [fm]	Rw [fm]
1	21.96	0.84	1.42	5.61	1.48	0.41	0.58	51	31	6.50	6.04
2	41.17	0.69	1.90	3.58	1.10	0.43	0.76	48	46	4.98	7.97
3	301.4 0	0.30	1.35	11.30	1.32	0.49	0.99	134	54	5.02	5.83

Table I. The best fit parameters of the Woods-Saxon optical model potential obtained from the analysis of the elastic scattering data for ${}^{26}Mg+{}^{13}C$.

In the Table, $r_v(r_w)$ and $a_v(a_w)$ are the real (imaginary) nuclear radius parameter and the surface diffuseness respectively. The Coulomb radius parameter is fixed to $r_{CO} = 1$ fm. \mathcal{X}^2 is given by

$$\chi^{2} = \frac{1}{N - f} \sum_{i} \frac{(\sigma_{exp}(\theta_{i}) - \sigma_{th}(\theta_{i}))^{2}}{(\Delta \sigma_{exp}(\theta_{i}))^{2}}$$

where N is the number of data points, f is the number of free parameters. J_v and J_w are the volume integral per interacting nucleon pair of the real and imaginary part, respectively.

$$I = f_{ij} + f_{ij} = -\left(\frac{1}{A_{ij}A_{jj}}\right) 4\pi \int_{0}^{\infty} (V(\mathbf{r}) + iW(\mathbf{r})) I^{0} d\mathbf{r}$$

The root mean square radius of the real and imaginary potentials are respectively given by

$$< R_{v}^{-2} > = \int_{0}^{\infty} V(r) r^{4} dr \Big/ \int_{0}^{\infty} V(r) r^{2} dr \qquad i < R_{w}^{-2} > = \int_{0}^{\infty} W(r) r^{4} dr \Big/ \int_{0}^{\infty} W(r) r^{2} dr$$

As shown in Fig. 2, all calculated optical model potentials fit quite well up to the point at which data ended. Due to the limited experimental data available, this experiment will be repeated in the near future to obtain additional data at larger angles to avoid the ambiguity in determination of optical potential that were needed for DWBA calculations. Also, repeating the experiment will allow us to accurately determine the ANC for the ${}^{26}Mg + n \rightarrow {}^{27}Mg$ systems.



FIG. 2. Optical model potential parameters for 12 MeV/nucleon ²⁶Mg elastic on ¹³C. Experimental points are the black dots.

In addition to the analysis with Woods-Saxon type potentials, the data have been analyzed using double folding potentials with the computer code OPTIMINIX^[5]. In this model, the potential is obtained by considering the effective NN(nucleon-nucleon) interaction between the matter distributions of the colliding particles.

$$v_{fold}(r) = \iint d\overline{r_1}^* d\overline{r_2}^* \rho_1(r_1) \rho_2(r_2) v_{eff}(\overline{r_1}^* + \overline{r} - \overline{r_2}^*).$$

The Hartree-Fock procedure was followed to calculate the nuclear density distributions, then the Jeukenne, Lejeune and Mahaux (JLM) effective interaction was used for the nucleon-nucleon interaction potential (V_{eff}). The resulting double folding potential is

$$U(r) = N_{\nu}N_{fald}(r) + tN_{\mu}N_{fald}(r)$$

The fitting procedure started with the average values $N_v=0.37$, $N_w=1.0$ for the renormalizations, and the standard range parameters $t_v=1.2$ fm and $t_w=1.75$ fm. The elastic scattering data is fitted by adjusting four parameters. The best fit to the data was obtained with $N_v=0.4$, $N_w=1.0$, $t_v=1.2$ and $t_w=1.75$ and is shown in Fig. 3.



FIG. 3. The blue line is the optical model calculations obtained with the double folding procedure. Experimental points are the black dots.

- M. McCleskey et al., *Progress in Research*, Cyclotron Institute, Texas A&M University (2009-2010) p. I-31.
- [2] A. Spiridon, R. Chyzh, M. Dag, M. McCleskey, and R.E. Tribble, *Progress in Research*, Cyclotron Institute, Texas A&M University (2012-2013) p.IV-50
- [3] L. Trache, A. Azhari, H.L. Clark, C.A. Gagliardi, Y.-W.Lui, A.M. Mukhamedzhanov, and R.E. Tribble, Phys. Rev. C 61, 024612 (2000).
- [4] M.H. Macfarlane, S.C. Piper, and M. Macfarlane, computer code PTOLEMY, Argonne National Lab Report ANL-76-11-rev-1 (1978).
- [5] F. Carstoiu, computer code OPTIMINIX (unpublished), Institute of Physics and Nuclear Engineering, Bucharest, Romania (1996).